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SOLUTION BY ROGER A. JOHNSON, Western Reserve University.

Let  $x = \angle EBG = \angle FBD$ ,  $y = \angle GBF$ , and  $\alpha = \angle EBD$ . Then  $\angle BDF = 150^\circ - \alpha/2$ , and  $\angle BGF = 90^\circ - y/2$ . Applying the law of sines,

$$\frac{\sin x}{\sin \left(150 - \frac{\alpha}{2}\right)} = \frac{FD}{BF}, \quad \frac{\sin y}{\sin \left(90^\circ - \frac{y}{2}\right)} = \frac{GF}{BF};$$

whence

$$\frac{\sin x}{\sin y} = \frac{\sin \left(150^\circ - \frac{\alpha}{2}\right)}{\sin \left(90^\circ - \frac{y}{2}\right)}. \quad (1)$$

Now in order that  $x = y$ , we must have either

$$150^\circ - \frac{\alpha}{2} = 90^\circ - \frac{y}{2}, \quad (a)$$

or

$$150^\circ - \frac{\alpha}{2} = 180^\circ - \left(90^\circ - \frac{y}{2}\right). \quad (b)$$

Letting  $y = \alpha/3$ , we obtain (a)  $\alpha = 180^\circ$ , (b)  $\alpha = 90^\circ$ , as the only values of  $\alpha$  for which this construction effects the trisection. Conversely, it is very easily seen directly that either a right angle or a straight angle is actually trisected.

To determine roughly the magnitude of the error for angles in general, we may safely replace  $\sin(90^\circ - y/2)$  by  $\sin(90^\circ - \alpha/6)$ ; we have then

$$\frac{\sin x}{\sin y} = \frac{\sin \left(30^\circ + \frac{\alpha}{2}\right)}{\sin \left(90^\circ - \frac{\alpha}{6}\right)}. \quad (2)$$

For small angles, the construction is far from accurate; in fact,

$$\lim_{\alpha \rightarrow 0} \frac{\sin x}{\sin y} = \frac{\sin 30^\circ}{\sin 90^\circ} = \frac{1}{2},$$

so that if  $\alpha$  is small, it is divided approximately in *fourths*. We find roughly, when  $\alpha = 30^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ$ , and  $180^\circ$ ,  $(\sin x)/(\sin y) = .710, .879, 1.000, 1.064, 1.066$ , and  $1.000$ , respectively. Again, by setting  $x = \alpha/2 - y/2$  in (1) and solving for  $y/2$ , we find

$$\cot \frac{y}{2} = 2 \cot \frac{\alpha}{2} + \sqrt{3};$$

whence we may compute the value of  $y$  corresponding to any value of  $\alpha$ . For example, if  $\alpha = 10^\circ$ ,  $y = 4^\circ 40'$  and  $x = 2^\circ 40'$ ; if  $\alpha = 60^\circ$ ,  $y = 21^\circ 46'$  and  $x = 19^\circ 7'$ ; if  $\alpha = 135^\circ$ ,  $y = 42^\circ 41'$  and  $x = 46^\circ 10'$ .

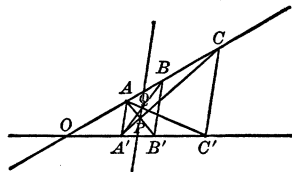
Also solved by HORACE OLSON.

#### 500. Proposed by R. T. MCGREGOR, Bangor, California.

$OABC$ ,  $OA'B'C'$  are two straight lines such that  $AA'$ ,  $BB'$ ,  $CC'$  are parallel.  $AB'$ ,  $A'C$  meet in  $P$ ;  $A'B$  and  $AC'$  meet in  $Q$ . Show by synthetic projective geometry that  $PQ$  is parallel to  $AA'$ . Milne's *Projective Geometry*, Chap. I, Ex. 20.

SOLUTION BY CLARIBEL KENDALL, University of Colorado.

Consider  $A, B, C$  as the first, third and fifth sides, respectively of a hexagon; and  $A', B', C'$  as the fourth, second, and sixth sides, respectively, of the same hexagon. (12), (45); (34), (61) meet in  $P$  and  $Q$ , respectively; hence, (23), (56) must meet in a point collinear with  $P$  and  $Q$ .



This follows as a special case of Pascal's Theorem for a hexagon inscribed in a conic.\* The conic is in this case degenerate, two straight lines. But (23), (56) are parallel and hence must have an infinitely distant point in common with  $PQ$ . Therefore  $PQ$  is parallel to  $AA'$ .

Also solved by MARJORIE L. BROWN, O. S. ADAMS (two methods), F. E. WOOD, NATHAN ALTSHILLER, HANNAH SUFFIN, and H. H. CONWELL.

### CALCULUS.

#### 416. Proposed by CHARLES N. SCHMALL, New York City.

If  $A$  be a point on a cycloid and  $C$  the corresponding position of the center of the generating circle, show that  $AC$  envelops another cycloid half the size of the first.

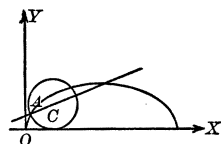
SOLUTION BY A. M. HARDING, University of Arkansas.

The coördinates of any point on the cycloid are  $x = a\theta - a \sin \theta$ ,  $y = a - a \cos \theta$ , and the coördinates of the center of the generating circle are given by  $x = a\theta$ ,  $y = a$ . The equation of  $AC$  is

$$\frac{x - a\theta}{\sin \theta} = \frac{y - a}{\cos \theta},$$

or

$$y - a = \cot \theta (x - a\theta).$$



The equation of a cycloid half the size of the given cycloid and having a cusp at  $O$  is

$$x = \frac{a\varphi}{2} - \frac{a}{2} \sin \varphi, \quad y = \frac{a}{2} - \frac{a}{2} \cos \varphi.$$

We propose to show that  $AC$  is always tangent to this cycloid.

The equation of any tangent to this cycloid is

$$y - \frac{a}{2} (1 - \cos \varphi) = \cot \frac{\varphi}{2} \left[ x - \frac{a}{2} (\varphi - \sin \varphi) \right].$$

Let  $\varphi = 2\theta$ . This equation then becomes

$$y - a \sin^2 \theta = \cot \theta [x - a\theta + a \sin \theta \cos \theta]$$

or

$$y - a = \cot \theta (x - a\theta),$$

which is the same as the equation of  $AC$ .

Also solved by C. N. SCHMALL, ELIJAH SWIFT, G. W. HARTWELL, HORACE OLSON, O. S. ADAMS, M. R. GAFFET, R. H. HOWARD, J. B. REYNOLDS, and SHIMPEI NISHIMURA.

#### 417. Proposed by H. S. UHLER, Yale University.

To the degree of approximation indicated show that  $(\sqrt{-1})^{\sqrt{-1}} = 0.207879576351$ .

SOLUTION BY WILLIAM HOOVER, Columbus, Ohio.

It is not difficult to show, as required in Todhunter's *Plane Trigonometry*, Ed. 1913, pp. 320-21, Examples 266, 275, "that  $(a + bi)^{a + \beta i}$  will be wholly real or imaginary if

$$(\beta/2) \log (a^2 + b^2) + \alpha \tan^{-1} (b/a)$$

is (I) zero, or an even multiple of  $\pi/2$ ; or (II) an odd multiple  $\pi/2$ . In the problem,  $a = \alpha = 0$ , and  $b = \beta = 1$ , the conditions corresponding to (I), 0 being called an even number.  $(\sqrt{-1})^{\sqrt{-1}} = e^{-\frac{1}{2}i\pi}$ , and the numerical value can be tested by using enough decimals in the values of  $e$  and  $\pi$ .

\* Cremona, *Elements of Projective Geometry*, translated by Leudensdorf, Articles 88 and 153, 3d edition.